1 Introduction

The increasing demand for miniature components (under 10 mm) with micro-scaled features (between tens and hundreds of microns) has spurred a significant amount of R&D work in the area of micro/meso-scale machine tools (mMTs) [1–3]. This development is motivated by a number of factors including: favorable scaling laws, lower machine cost, and increased machine portability. mMT test beds developed by research groups are orders of magnitude smaller, with typical sizes smaller than 300 mm × 300 mm × 300 mm, and several times less expensive than their traditional precision machine tool counterparts. Some newer commercial precision machines are also showing a trend of smaller size and working volume to address part miniaturization. The Kern Micro Machine Tabletop has a working volume of 200 mm × 150 mm × 150 mm [4]. The Makino HYPER2J has a working volume of 200 mm × 150 mm × 150 mm [5].

Since the introduction of Japan’s Mechanical Engineering Laboratory microfactory concept in 1998 [6], several mMTs have been developed. These machines include three-axis milling machines developed by Vogler et al. [1] and Subrahmanian and Ehmann [2], and a five-axis machine developed by Werkmeister and Slocum [3]. Precision commercial machines have also been developed to address part miniaturization. The Kern Micro Machine Tabletop has a working volume of 160 mm × 100 mm × 200 mm [4]. The Makino HYPER2J has a working volume of 200 mm × 150 mm × 150 mm [5].

Accuracy in a machine tool is derived from two sources: mechanical precision and software-based compensation. One goal of mMT development is to achieve relative machining accuracies of 10⁻³–10⁻⁵. For a 1 mm feature, this translates into an absolute accuracy of 1.0–0.01 μm. It is impractical to build a machine with this level of inherent accuracy. A superior approach, now common for many types of machine tools and robotic manipulators, is to build a highly repeatable machine with an easily achievable level of inherent accuracy and then use calibration methods to improve positioning accuracy by compensating for errors [7]. Since the marginal cost of mechanical component and assembly accuracies increases sharply as accuracy level increases, calibration becomes increasingly important as the target accuracy for a machine increases. Thus, calibration equipment and methodologies are arguably more essential for mMTs than for traditional machine tools and manipulators.

A large body of work exists for calibration of conventional machine tools and robotic manipulators. The calibration process typically consists of four steps: modeling, measurement, parameter estimation, and implementation [7]. Numerous kinematic and thermal error models have been proposed [7–11]. Similarly, numerous measurement devices and methodologies have been developed, including laser interferometers, fixed and telescoping ball bars, and trigger probes, to name a few [12–26]. Researchers have developed several techniques to estimate error model parameters from measurements including direct measurement, linear and nonlinear least squares estimation, and L∞ estimation [27,28]. Finally, compensation is implemented using various techniques, including lookup tables and inverse kinematic models.

The shortcomings of the existing calibration work applied to mMTs occur primarily in the measurement step. Laser interferometer systems are too costly, not easily portable, and not suited for frequent recalibration of a machine. No on-machine measurement equipment currently exists that is small and accurate enough for mMT requirements. A small telescoping ball bar could be constructed, but it would suffer from small displacement range and any motion inaccuracies due to miniaturizing the telescoping mechanism. Also, the small sizes involved would make changing the base positions of the miniature ball bar (for triangulation) tedious. Conventional coordinate-measuring machine (CMM) trigger probes are too large to mount in an mMT spindle. Position sensitive detectors (PSDs), which have been used for calibration of micro-positioning stages, currently have resolution limitations of greater than 1 μm for devices with enough area to cover a 25 mm × 25 mm mMT working envelope [29]. Since PSD resolution improves greatly as working envelope decreases, these devices are suitable for calibration applications with a smaller working envelope.
This paper focuses on the development, analysis, and application of a new calibration methodology and equipment specifically suited for mMTs. The measurement system is consistent with the value characteristics of mMTs, including small size, low cost, portability, and high accuracy. A measurement methodology is developed to take advantage of unique mMT attributes, including small travel range and highly repeatable mounts, and to address the need for frequent recalibration of mMTs. Along with the measurement system, the methodology provides a means for rapid mMT recalibration by reducing the number of repeated measurements. This paper first discusses the design of the error measuring system. Next, a hybrid calibration methodology is presented. Validation is then conducted by applying the measurement system and methodology to experimental calibration of a three-axis mMT. Finally, important measurement system design factors are analyzed using simulation.

2 Design of Measurement System

A contact trigger probe system has been developed to address the requirements for mMT calibration. This system can be designed small enough to mount on an mMT. It also can attach quickly and easily to the machine, since the sensor and stylus mount to the machine in the same manner as a workpiece and cutting tool, respectively. This system detects contact between the stylus and sensor assembly. Measurements are obtained by reading the machine axis positions at the contact point.

The sensor assembly consists of a precision plane artifact, a 1-degree-of-freedom (DOF) flexure, a load cell for detecting contact force, and a kinematic coupling for machine attachment. This arrangement addresses the size drawback of conventional CMM probes by separating the sensor and the stylus. Figure 1 shows the layout of this trigger probe. The kinematic coupling shown in Fig. 1 is tailored to work with the mMT being calibrated. The kinematic coupling used allows for three artifact orientations by simply rotating the coupling.

The goal of the measurement system is to accurately and repeatably detect small contact forces between the stylus and the planar artifact. This goal is accomplished by implementing a compression load cell and utilizing a specialized notch-type linear flexure design. The flexure provides flexibility in one degree-of-freedom coaxial to the load cell sensing direction, while remaining rigid in all other degrees of freedom. Figure 2 shows detail of the flexure design.

The stiffness of the flexure, \( K \), in the flexible direction is given by Smith and Chetwynd as [30]

\[
K = \frac{8Ebr^{5/2}}{9\pi L^{17/2}}
\]

where \( E \) is the elastic modulus, and \( b, t, L, \) and \( R \) represent the geometry of the flexure, as shown in Fig. 4. The geometry for this application is chosen based on a desired stiffness, geometric constraints, and machinability requirements. The stiffness should be chosen to be roughly an order of magnitude smaller than the combined stiffness of the machine structural loop to provide adequate force-sensing sensitivity.

The inclination angle of the flexure, \( \alpha \), is also an important design parameter. The measurement system detects the point of contact between the stylus and the artifact. This measurement is equivalent to a zero-distance measurement in the plane normal direction. Since it is desirable to have an orthogonal measurement space, \( \alpha \) should be selected so that the plane normals for the artifact orientations are orthogonal. Using a ball and vee-groove-type kinematic coupling the plane unit normals, \( \hat{n}_i \), for the three orientations are given by (assuming a convenient first orientation in the \( x-z \) plane)

\[
\hat{n}_1 = \langle \cos \alpha, 0, \sin \alpha \rangle
\]

\[
\hat{n}_2 = \langle \cos \beta_2 \cos \alpha, \sin \beta_2 \cos \alpha, \sin \alpha \rangle
\]

\[
\hat{n}_3 = \langle \cos \beta_3 \cos \alpha, \sin \beta_3 \cos \alpha, \sin \alpha \rangle
\]

where \( \beta_2 \) and \( \beta_3 \) are the \( z \)-axis rotation angles for the second and third orientations, respectively. Equation (2) is used to find an \( \alpha \) that satisfies the condition

\[
\hat{n}_1 \cdot \hat{n}_2 = \hat{n}_2 \cdot \hat{n}_3 = \hat{n}_3 \cdot \hat{n}_1 = 0
\]

For example, \( \alpha = 54.7 \) deg for the kinematic coupling shown in Fig. 3, which has three orientations spaced at 120 deg (\( \beta_2 = 120 \) deg, \( \beta_3 = 240 \) deg).

A precision planar artifact is rigidly fixed to the inclined surface of the flexure. This artifact forms the surface on which the contact force takes place. This geometry of artifact is a good choice for mMTs, due to the availability of small planes with high flatness tolerances (i.e., optical windows, mirrors, gauge blocks, optical windows, mirrors, gauge blocks, etc.).
etc.). In comparison, the use of planar artifacts for large volume machine tools is much less feasible due to the rapidly increasing cost of producing and maintaining large diameter flat surfaces.

3 Hybrid Calibration Methodology

The measurement method is a hybrid technique that uses both off-machine measurements and on-machine measurements to estimate kinematic error parameters. The off-machine measurements are used to determine the orientation relationships among the artifact setups, which is important for kinematic parameter estimation. The on-machine measurements are made using the measurement system described in Sec. 2. This methodology is advantageous for frequent recalibration since the number of on-machine measurements is reduced. This reduction results from not needing to repeat off-machine measurements for each recalibration; they only need to be repeated periodically to ensure that artifact changes over time do not affect the calibration accuracy. Since the measurement system can be set up very quickly, on-machine measurement time is reduced. Figure 4 summarizes the hybrid calibration methodology.

This methodology is similar to existing methodologies in that an artifact is used along with a measurement system and machine axis positions to measure kinematic errors. However, it is differentiated by the type of artifact used, the use of off- and on-machine measurements, and the ease of repositioning of the artifact. For example, a telescoping ball bar measures errors along the axis of the bar, while the method described here measures errors in the direction normal to the planar artifact. Also, the planar artifact requires less repositioning to make error measurements throughout the entire machine working volume than a ball bar. Furthermore, the planar artifact is fast and easy to reposition since it mounts to the machine using a kinematic coupling. This methodology is differentiated from a laser interferometer based approach by the on-machine nature of the measurement system and ease of machine recalibration. Due to its size, a laser interferometer cannot be measured on the CMM, and the orientation of each pallet/coupling base combination can be measured on the CMM. Alternatively, the orientation of each coupling base can be measured on the CMM, and the orientation of each pallet can be measured on the machine after it has been calibrated with respect to the coupling base coordinate system.

Using the measured points for each given orientation, the normalized least-squares planes are determined such as a CMM, since these measurements are used as a reference for the on-machine measurements.

The first step in off-machine measurements is to measure the position and orientation of the pallet. Next, keeping the coupling base stationary, the pallet is removed and the artifact is coupled to the base in a particular orientation. The artifact orientation is then measured. Since a least-squares plane will be fitted to each orientation data set, at least 25 well-distributed measurement points are preferred to achieve a good fit. The artifact measurements are repeated for each orientation.

If more than one pallet or coupling base is used on a machine (or manufacturing cell), two methods of relating artifact orientations to pallet orientations are possible. First, the position and orientation of each pallet/coupling base combination can be measured on the CMM. Alternatively, the orientation of each coupling base can be measured on the CMM, and the orientation of each pallet can be measured on the machine after it has been calibrated with respect to the coupling base coordinate system.

Using the measured points for each given orientation, the normalized least-squares planes are determined

$$A_i x + B_i y + C_i z + D_i = 0$$

where $x$, $y$, and $z$ represent the measurement system coordinate space, $A$, $B$, $C$, and $D$ are parameters, and the subscript $i$ corresponds to the orientation. The quality of the fit can be determined by studying the residuals

$$r_{ij} = z_{ij} + \frac{A_{ij} x_i + B_{ij} y_i + D_i}{C_i}$$

where the subscript $j$ corresponds to the measurement number.

3.2 Artifact Orientation Relationships. The orientation relationships among the setups are determined using off-machine measurements of the three artifact orientations. The submicron repeatability of the mMT kinematic couplings ensures that the relationships among the orientations will be repeatable between the off-machine and on-machine measurement setups.

Using the fitted planes from Eq. (4), coordinate systems for the workpiece mount and artifact orientations are assigned. Next, the rigid body transformations from each artifact coordinate system to the workpiece coordinate system are determined. These transformations allow on-machine measurements to be expressed in terms of the workpiece coordinate systems.

First, coordinate systems ($CS_1, CS_2, CS_3$) are defined for each of the three artifact orientations. Figure 5 shows the geometry discussed in this section. Each of these frames is defined to have its origin at the intersection of the three planes, $x_0$. The $x$-axis unit normal of $CS_1$ is defined to be
where \( \hat{n}_i \) is the unit normal of plane \( i \). The \( z \)-axis of \( \text{CS}_1 \) is defined by \( \hat{n}_1 \), and the \( y \)-axis completes a right-hand coordinate system. \( \text{CS}_2 \) is defined to have the same \( x \)-axis as \( \text{CS}_1 \), with the \( z \)-axis defined by \( \hat{n}_2 \). The \( y \)-axis of \( \text{CS}_2 \) completes the right-hand coordinate system. The \( \text{CS}_3 \) \( x \)-axis is defined by \( \hat{n}_3 \), following the same convention as in Eq. (6). The \( z \)-axis is defined by \( \hat{n}_3 \), and the \( y \)-axis again completes the right-hand coordinate system.

Given these coordinate system definitions, coordinates \( r_i \) in \( \text{CS}_2 \) and \( \text{CS}_3 \) are mapped to \( r_1 \) in \( \text{CS}_1 \) by

\[
T_1 = \begin{bmatrix} \hat{n}_1 & \hat{n}_2 & \hat{n}_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

where \( T_i \) is determined in the following manner. It is apparent from the coordinate system definitions that \( T_2 \) is simply \( x \)-axis rotation of \( \theta_{21} \), where

\[
\theta_{21} = \cos^{-1}(\hat{n}_2 \cdot \hat{n}_1)
\]

\( T_3 \) is accomplished by two successive rotations

\[
T_3 = \text{rotate}(z, \phi_{31}) \text{rotate}(x, \theta_{31})
\]

where \( \phi_{31} \) is the angle between \( \hat{n}_{31} \) and \( \hat{n}_1 \), and \( \theta_{31} \) is the angle between \( \hat{n}_1 \) and \( \hat{n}_1 \).

Since measurements are all taken in the CMM reference frame, we need a final transformation from \( \text{CS}_1 \) coordinates to CMM (\( \text{CS}_0 \)) frame coordinates. This transformation appears naturally from the definition of \( \text{CS}_1 \) as

\[
\begin{array}{c}
\theta_{10} = \begin{bmatrix} \hat{n}_{12} \times \hat{n}_{21} & \hat{n}_{12} & \hat{n}_{31} \end{bmatrix} \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
\]

Thus, the orientation relationships are known, and any point in a coordinate system \( \text{CS}_1 \) can be transformed to its point in \( \text{CS}_0 \) by

\[
T_0 = T_1 T_i r_i
\]

For the parameter estimation step, the inverse orientation relationships are used. That is, \( T_0 \) and \( T_1 \) need to be calculated. For this application, \( T_0 \) and \( T_1 \) simply equal \( T_1^{-1} \) and \( T_1^{-1} \).

The procedure above is also used to determine \( T_{n+1} \), the transformation from workpiece coordinates to the world (CMM) coordinates. This reference frame is defined by the measurements of three planes of the mMT workpiece mount. Knowing these transforms, the relationship between each artifact orientation and the workpiece coordinates is established.

### 3.3 On-Machine Measurements

The on-machine measurements are obtained using the trigger probe described in Sec. 2. The purpose of this procedure is to gather the data needed to estimate the machine kinematic error parameters. Measurements are taken throughout the working volume for each of three artifact orientations. For these measurements, the trigger probe sensor is mounted to the mMT workpiece table using a kinematic coupling, and the stylus is mounted in the spindle, as shown in Fig. 6. A measurement is taken by moving the machine to a commanded \( x \)-\( y \) position, with the \( z \)-axis sufficiently retracted, and then moving the stylus toward the sensor in the \( z \)-direction. The axis readings at the point of contact are recorded by the controller. Measurements are taken throughout the \( x \)-\( y \) envelope to obtain representative data for the machine. This process is repeated for several (minimum of three) orientations of the trigger probe sensor on the kinematic coupling. For example, three artifact orientations are shown on a machine in Fig. 7. For a four- or five-axis machine, the on-machine measurements need to be repeated with a different stylus set length in order to observe tool orientation parameters.

The on-machine measurement process requires a simple modification for mMTs that do not use ball and vee-groove kinematic couplings for workpiece mounting. This modification is the addition of an interface coupling between the machine mount and the measurement system. This coupling has a ball and vee-groove kinematic coupling on one side and a means for machine attachment on the other side.

### 3.4 Kinematic Error Modeling

In order to apply the off-machine and on-machine measurements to the calibration of an mMT, proper kinematic modeling and error parameter estimation methods must be employed.

Kinematic modeling relates the pose of the tool tip to the machine axes positions. Models can include error parameters that are estimated during the calibration process. The zero reference model described by Mooring et al. [7] will be used here to model a three-axis mMT. This convention is convenient for the modeling of manipulators whose principal axes are nominally aligned with the Cartesian coordinate axes, as is the case in most machine tools.

The zero-order error vector (the error parameters are not functions of joint positions), \( \rho \), for a three-axis mMT is comprised of the parameters shown in Table 1, where \( E \) is an error matrix, \( e \) is a small angular error, \( \delta \) is a displacement error, and the subscripts \( x, y, z, \) and \( t \) correspond to the Cartesian axes and tool offset. This

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Error parameters for three-axis mMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error transform</td>
<td>Parameters</td>
</tr>
<tr>
<td>( Ey )</td>
<td>( e_{y}(\text{rad}) )</td>
</tr>
<tr>
<td>( E_{yx} )</td>
<td>( e_{xy}(\text{rad}) )</td>
</tr>
<tr>
<td>( E_{yz} )</td>
<td>( e_{yz}(\text{rad}) )</td>
</tr>
<tr>
<td>( Ex )</td>
<td>( e_{x}(\text{rad}) )</td>
</tr>
<tr>
<td>( E_{zx} )</td>
<td>( e_{zx}(\text{rad}) )</td>
</tr>
<tr>
<td>( E_{zy} )</td>
<td>( e_{zy}(\text{rad}) )</td>
</tr>
<tr>
<td>( \delta_x(\text{mm}) )</td>
<td></td>
</tr>
<tr>
<td>( \delta_y(\text{mm}) )</td>
<td></td>
</tr>
<tr>
<td>( \delta_z(\text{mm}) )</td>
<td></td>
</tr>
</tbody>
</table>
model can be extended if necessary to include higher-order error parameters, as described in Freeman [31]. The measured error, \( e_{ij} \), for an artifact orientation \( i \) and measurement number \( j \), is defined as the \( z \)-axis position of the measured point in the appropriate calibration plane coordinate system (a \( z \)-axis position of zero lies exactly on the calibration plane). This error is determined by substituting the measured axis positions into the kinematic model.

3.5 Estimation of Kinematic Parameters. The goal of the estimation process is to determine the kinematic parameters that best account for the measured positioning errors. For this application, both \( L_2 \) (least squares) and \( L_\infty \) (worst case) minimization methods are used. \( L_2 \) parameter estimation is described in Zhuang and Roth [32]. This procedure is an iterative method that estimates the kinematic parameter vector, \( p \), that minimizes measurement errors in a least-squares sense. \( L_\infty \) parameter estimation, described by Tajbaksh et al. [28], minimizes the maximum error using a linear programming approach. Other estimation methods, such as implicit-loop methods [26], are compatible with this hybrid methodology and can be used if desired.

4 Validation of Methodology

The measurement system and associated methodology described in Secs. 2 and 3 are now analyzed and validated. First, the measurement error of a fabricated measurement system is predicted and validated. Next, the parameter estimation results of experimental calibration using three artifact orientations are examined. Then, the effects of measurement parameters on estimation accuracy are analyzed using simulation. Finally, the calibration methodology is validated by comparing the accuracy of machined features before and after compensation.

4.1 Experimental Measurement System. The measurement system and associated methodology has been validated for a three-axis mMT test bed developed at the University of Illinois at Urbana-Champaign (UIUC) and is shown in Fig. 8. This machine was developed to address some of the limitations of the first generation testbed developed by Vogler et al. [1], including low stiffness and workpiece mounting difficulty. The stiffness was improved by using symmetrically positioned moving coil actuators and linear ways. Each axis is capable of \( 5 \) g accelerations. The machine has a work envelope of \( 25 \text{ mm} \times 25 \text{ mm} \times 25 \text{ mm} \) and an overall size of \( 350 \text{ mm} \times 300 \text{ mm} \times 205 \text{ mm} \). Position feedback is obtained using optical encoders with \( 0.1 \mu \text{m} \) resolution. The spindle is air driven and air bearing supported with a maximum speed of \( 160,000 \) rpm. Symmetrically positioned linear ball-bearing guides are used for the linear motion constraint of each axis. Workpiece and spindle mounting are simplified through the use of magnetically preloaded kinematic couplings. Kinematic couplings provide exact constraint to the mounted component, resulting in submicron repeatability [33]. In addition to high repeatability, kinematic couplings have been developed that enable high long-term accuracy for multi-pallet and multi-machine systems [34,35]. These couplings are beneficial for mMTs in that they provide a rapid means to change spindles and workpieces, and allow a workpiece to be removed midprocess for offline inspection. Figure 8 shows the kinematic coupling between the workpiece mount and the machine.

The measurement system is fabricated using the design described in Sec. 2. The linear flexure is designed using Eq. (1) to achieve a stiffness of \( 0.02 \) N/\( \mu \text{m} \). This low stiffness enables high sensitivity of the force measurement. Table 2 shows the parameters of the flexure design that were shown graphically in Fig. 2. A Honeywell Sensotec Model 13 compression load cell is used to detect contact force between the stylus and artifact.

The inclination angle, \( \alpha \), of the artifact is found to be \( 54.7 \) deg using Eqs. (2) and (3). A 50-mm-diam precision optical mirror is used for the planar artifact surface, providing a flatter surface for measurements than a ground or polished metal plate. The stylus has a 3.0 mm shank for mounting in the mMT spindle and has a 3.0-mm-diam spherical ruby tip, which provides an accurate surface for probing.

A microcontroller is used to automatically set a threshold based on the nominal force sensor reading. This implementation also allows the threshold sensitivity to be adjusted for maximum performance and reliability. A comparator is used to detect the probe contact and send a 24 V logic signal to the machine controller. For safety, a second comparator is also used to detect the fault condition of a contact force greater than \( 5 \) N. The logic signal sent by this comparator disables the machine axes to prevent damage to the machine or calibration equipment.

4.2 Predicted Measurement Error. Before using the experimental measurement system described in Sec. 4.1 for mMT calibration, the measurement error of the system is analyzed. This analysis is important to ensure the appropriate calibration accuracy and should be repeated for each mMT it will be used on since measurement error is, in part, machine dependent.

Measurement error is analyzed by examining error sources in the measurement process. The measurement process is as follows: the machine is moved to a desired \( x-y \) position, and then the \( z \)-axis is advanced until contact is detected, as depicted in Fig. 9. The sensor assembly is repositioned between sets of measurements to obtain several data sets. Therefore, the measurement error is driven by deviations associated with the measurement system, \( \sigma_m \), and deviations associated with repositioning of the sensor, \( \sigma_r \). Assuming the error sources are uncorrelated, the measurement error in the \( z \)-axis direction, \( \sigma_z \), is given by the additive law of variances.

### Table 2 Flexure design specifications

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>0.021</td>
<td>N/( \mu \text{m} )</td>
</tr>
<tr>
<td>( E )</td>
<td>70</td>
<td>GPa</td>
</tr>
<tr>
<td>( b )</td>
<td>38.1</td>
<td>mm</td>
</tr>
<tr>
<td>( t )</td>
<td>0.127</td>
<td>mm</td>
</tr>
<tr>
<td>( L )</td>
<td>12.7</td>
<td>mm</td>
</tr>
<tr>
<td>( R )</td>
<td>1.588</td>
<td>mm</td>
</tr>
</tbody>
</table>

Fig. 8 UIUC three-axis mMT test bed
These two terms can be expanded to show individual component error contributions

\[
\sigma_m = \sqrt{\left(\sigma_f^2 + \sigma_r^2 + \sigma_{dl}^2 + \sigma_{rl}^2 + \sigma_{dt}^2 + \sigma_{rt}^2 + \sigma_{rs}^2 + \sigma_{ts}^2 + \sigma_{tr}^2 + \sigma_{ts}^2\right)}
\]

where

- \(\sigma_f\): z-axis component of flatness error of calibration plane
- \(\sigma_r\): z-axis component of roundness error of stylus tip
- \(\sigma_{dl}\): z-axis component of combined lateral deflection due to contact measurement
- \(\sigma_{rl}\): z-axis component of lateral (x-y) positioning repeatability of machine
- \(\sigma_{da}\): combined axial deflection due to contact measurement
- \(\sigma_{ta}\): error caused by position capture time lag
- \(\sigma_{ra}\): axial (z-axis) positioning repeatability of machine
- \(\sigma_{tc}\): kinematic coupling repeatability in probing direction.

The z-axis components of \(\sigma_f, \sigma_r, \sigma_{da}\), and \(\sigma_{rl}\) are determined by analyzing the direction of the error contribution in relation to the direction of measurement as shown in Fig. 9. These components are easily determined using the inclination angle \(\alpha\)

\[
\begin{align*}
\sigma_f &= \sigma_f \sin \alpha \\
\sigma_r &= \sigma_r \sin \alpha \\
\sigma_{da} &= \sigma_{da} \cos \alpha \\
\sigma_{rl} &= \sigma_{rl} \cos \alpha
\end{align*}
\]

Stylus run-out errors are negligible since the spindle is not allowed to rotate during the measurement process. This was accomplished for the experimental calibration by mechanically clamping the spindle in place using the pin used to hold the spindle stationary during tool change.

Estimation of the specific error components in Eqs. (13) and (14) is based on the hardware manufacturer’s specifications and experimental results and are shown in Table 3. From manufacturer specifications, the rated flatness of the optical mirror is 0.063 \(\mu m\), and the maximum diameter variation of the stylus tip (AFBMA Grade 5 ball) is 0.125 \(\mu m\). Assuming these ratings are three times the standard deviations, \(\sigma_f\) and \(\sigma_r\) are estimated to be 0.021 and 0.042 \(\mu m\), respectively. Time lag error is also estimated from manufacturer specifications

\[
\sigma_t = v_p \left(t_c + t_l\right)
\]

where \(v_p\) is the probing speed (0.1 mm/s), \(t_c\) is the comparator lag (2 \(\mu s\)), and \(t_l\) is the lag of the hardware-based controller trigger (100 \(\mu s\)).

The combined deflections are calculated using experimental values for triggering threshold (10 mN), combined lateral compliance (5 \(\mu m/N\)), and combined axial compliance (2 \(\mu m/N\)). Repeatability of a single machine axis was tested using a capacitance probe test. For this test, a specified location was approached 50 times from alternating directions, and the average capacitance sensor reading was recorded during a 100 ms dwell at the specified location. Kinematic coupling repeatability is estimated from measurements of the mating balls and vee-groove blocks of the coupling, since coupling repeatability has been shown to approach the surface roughness of the mating components [31]. The vee-groove block roughness (0.4 \(\mu m\ Ra\)) dominates the ball roughness (0.025 \(\mu m\ Ra\)) for this estimation. Using Eqs. (12)–(14) and individual error estimations in Table 3, \(\sigma_{fa}, \sigma_{ra}, \) and \(\sigma_r\) are predicted to be 0.066, 0.400, and 0.405 \(\mu m\), respectively. Thus, the ISO repeatability standard 3\(\sigma\) value of \(\sigma_r\) is 1.22 \(\mu m\).

### 4.3 Experimental Measurement Error

Two experiments were conducted to validate estimates of \(\sigma_{fa}\) and \(\sigma_{ra}\). \(\sigma_{fa}\) was validated by probing a single point 25 times with an approach speed of 0.2 mm/s. Three such tests were conducted in succession. Figure 10 shows that the measurement error had a maximum of 0.1 \(\mu m\) (one encoder count) and a standard deviation of 0.059 \(\mu m\). The \(\sigma_{ra}\) was validated by probing two points ten times each, with the kinematic coupling being detached after each measurement; \(\sigma_r\) was calculated to be 0.58 \(\mu m\) from the measured errors shown in Fig. 11.

Table 4 compares the predicted and experimental values for \(\sigma_{fa}\) and \(\sigma_{ra}\). The predicted and experimental values for \(\sigma_{ra}\) show good agreement. The experimental value of \(\sigma_{ra}\) is somewhat higher than predicted. This discrepancy can be attributed to the amplification of the angular coupling errors due to the distance between the calibration plane and the centroid of the kinematic coupling.

Recalling Eq. (12), the experimental value of \(\sigma_r\) is calculated to be 0.583 \(\mu m\). Thus, the goal of micron to submicron measurement accuracy is achieved. In addition, significant improvement of the

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**Table 3** Expected error component values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated ((\mu m))</th>
<th>Method of estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_f)</td>
<td>0.025</td>
<td>Manufacturer specification</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>0.042</td>
<td>Manufacturer specification</td>
</tr>
<tr>
<td>(\sigma_{da})</td>
<td>0.030</td>
<td>Experimental</td>
</tr>
<tr>
<td>(\sigma_{rl})</td>
<td>0.038</td>
<td>Experimental</td>
</tr>
<tr>
<td>(\sigma_{ta})</td>
<td>0.020</td>
<td>Experimental</td>
</tr>
<tr>
<td>(\sigma_{ra})</td>
<td>0.010</td>
<td>Manufacturer specification</td>
</tr>
<tr>
<td>(\sigma_{fa})</td>
<td>0.038</td>
<td>Experimental</td>
</tr>
<tr>
<td>(\sigma_{ra})</td>
<td>0.400</td>
<td>Experimental</td>
</tr>
</tbody>
</table>

Fig. 10 Repeatability results for a single point
measurement accuracy is possible by improving the repeatability of the coupling, since kinematic couplings have been shown to be repeatable to 0.05 μm [31].

4.4 Initial Calibration Results. The hybrid methodology developed in Sec. 3 was implemented for the calibration of the UIUC three-axis mMT. This implementation was used to test the efficacy and repeatability of the calibration methodology for both $L_2$ and $L_\infty$ parameter estimation methods.

Off-machine measurements of the artifact orientations were taken, as described in Sec. 3.1, using a Brown and Sharpe EXCEL CMM. Twenty-five measurements were taken in an evenly distributed square pattern for each of three artifact orientations. The fitted parameters found using Eq. (4) for each orientation are shown in Table 5.

On-machine measurements were taken with the measurement system described in Sec. 2 using the methodology described in Sec. 3.3. Two independent sets of 25 measurements were taken in a square pattern for each artifact orientation. These two data sets were used to test the repeatability of the results. Parameters were estimated with both $L_2$ and $L_\infty$ methods for both data sets using the zero-order error model parameters described in Sec. 3.4.

Table 6 compares the estimated error parameters using the $L_2$ and $L_\infty$ methods for both data sets using Eq. (4). The difference of parameter $\varepsilon_{\alpha}$ between the two data sets is 3.2% and 8.8% for $L_2$ and $L_\infty$ methods, respectively. However, the difference between the $L_2$ and $L_\infty$ methods for the same parameter is approximately 45%. This difference, which can be attributed to the different error weighting between $L_2$ and $L_\infty$ methods, suggests the presence of local minima that may be the result of inadequate measurement parameters (e.g., number of artifact orientations, number of measurements). Different measurement parameters may be necessary to produce robust and accurate parameter estimates.

4.5 Effect of Measurement Parameters on Estimation Accuracy. Simulations have been conducted to study methods for increasing estimation accuracy in the presence of measurement errors. These simulations examine the effect of the number of artifact orientations and the number of measurement points per orientation on parameter estimation accuracy.

The first set of simulations compares the estimation accuracies using three and six artifact orientations. First, the ability to correctly estimate a set of known error parameters was simulated for three orientations using the $L_2$ method. The error parameters in Table 6 were used. Figure 12 shows that the estimation accuracy deteriorates very quickly with measurement noise. For example, the maximum parameter estimation error is 200% with only $\sigma = 1.7$ μm of measurement noise. A second similar simulation was conducted using six artifact setups instead of the minimum three required. The second set of three artifact orientations is simply translated in the $z$-direction from the first set by 6 mm. It can be significantly different error parameter estimates. For example, the difference of parameter $\varepsilon_{\alpha}$ between the two data sets is 3.2% and 8.8% for $L_2$ and $L_\infty$ methods, respectively. However, the difference between the $L_2$ and $L_\infty$ methods for the same parameter is approximately 45%. This difference, which can be attributed to the different error weighting between $L_2$ and $L_\infty$ methods, suggests the presence of local minima that may be the result of inadequate measurement parameters (e.g., number of artifact orientations, number of measurements). Different measurement parameters may be necessary to produce robust and accurate parameter estimates.

![Fig. 11 Repeatability results for repositioning of sensor](image1)

![Fig. 12 $L_2$ estimation accuracy sensitivity to measurement noise](image2)
seen that the addition of extra artifact orientations greatly increases the estimation accuracy in the presence of noise. With \( \sigma = 3 \) \( \mu \)m of measurement noise, the maximum estimation error is below 20%. Extra artifact setups can be incorporated experimentally by the addition of a repeatable spacer coupling, multiple artifacts, changing the stylus length, or by implementing a distance sensor such as a laser or linear variable displacement transducer.

The second set of simulations examined the effect of the number of measurements on the estimation accuracy. Six artifact orientations and measurement errors of 5 \( \mu \)m were used. Figure 13 shows that the worst parameter error is reduced below 10% using 15 measurements per orientation. The maximum parameter error is only reduced slightly by using more than 15 measurements. Therefore, at least 15 measurements are desirable to achieve high calibration accuracy. These results show that the 25 measurements used for calibration in Sec. 4.4 are more than adequate.

### 4.6 Calibration Using Six Artifact Orientations

Given the importance of increasing the number of artifact orientations to parameter estimation accuracy, calibration of the UIUC three-axis mMT was repeated using six artifact orientations. These six orientations were achieved by taking this measurement for two sets of three artifact orientations. The probe set length was increased by 2.0 mm for the second set, creating a z-axis offset from the first set. All other measurement parameters, such as the number and pattern of measurements, were held the same as for the initial calibration.

Table 7 shows the estimated parameters using the \( L_2 \) and \( L_\infty \) methods. These parameters have much less variability between the two methods, indicating a more robust parameter estimate.

The reduction in each measured error, \( e_{ij} \), is predicted by implementing software compensation. This compensation uses the estimated error parameters in Table 7 in the zero-reference kinematic model discussed in Mooring et al. [7] to obtain the compensated measurement. Table 8 shows the measured error improvement statistics predicted for each estimation method. Each method captures more than 90% of the uncompensated machine measured error, reducing the measured error standard deviation to below 1.5 \( \mu \)m.

### 4.7 Validation of Calibration Methodology

Validation of the calibration methodology is conducted by comparing measured accuracies of machined features created before and after compensation. Compensation was achieved by implementing the actual machine inverse kinematics into the machine controller using an iterative method as described by Hollarbach et al. [36] and the error parameters shown in Table 7.

The chosen machined features are shown in Fig. 14. A series of 20 0.1 mm (nominal) deep slots with 1 mm spacing were machined along the x- and y-axes both with and without compensation. The depth accuracy of these slots is determined primarily by the first three error parameters from Table 7. Figure 15 shows the actual depth of these slots, measured using a Mitutoyo Contracer, with and without compensation. The maximum depth error was reduced from 2.01 to 0.08 \( \mu \)m/mm along the x-axis and from 1.83 to 0.08 \( \mu \)m/mm along the y-axis.

In addition, \( L \)-shaped pockets were machined before and after compensation to test the perpendicularity between the x- and y-axes, which corresponds to \( e_{xz} \). The perpendicularity of the pocket walls was measured on a Brown and Sharpe EXCEL CMM using a 0.75-mm-diam stylus. The measured perpendicularity error was reduced by the compensation from 0.074 to 0.01 deg. This angular improvement corresponds to a linear error improvement from 12.9 to 1.7 \( \mu \)m at a distance of

![Fig. 13 Effect of number of measurement points on estimation accuracy](image1.png)

![Fig. 14 Machined features for calibration validation](image2.png)

### Table 7 Comparison of estimated error parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( L_2 )</th>
<th>( L_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_{xy} ) (rad)</td>
<td>0.0035</td>
<td>0.0030</td>
</tr>
<tr>
<td>( e_{yz} ) (rad)</td>
<td>0.0054</td>
<td>0.0060</td>
</tr>
<tr>
<td>( e_{xz} ) (rad)</td>
<td>−0.0016</td>
<td>−0.0017</td>
</tr>
<tr>
<td>( e_{yx} ) (rad)</td>
<td>0.0027</td>
<td>0.0038</td>
</tr>
<tr>
<td>( e_{yx} ) (rad)</td>
<td>−0.0051</td>
<td>−0.0063</td>
</tr>
<tr>
<td>( e_{y=} ) (rad)</td>
<td>0.0030</td>
<td>0.0031</td>
</tr>
<tr>
<td>( \delta_{y} ) (mm)</td>
<td>0.0010</td>
<td>−0.0065</td>
</tr>
<tr>
<td>( \delta_{y} ) (mm)</td>
<td>0.0009</td>
<td>−0.0173</td>
</tr>
<tr>
<td>( \delta_{x} ) (mm)</td>
<td>−0.0111</td>
<td>−0.0218</td>
</tr>
</tbody>
</table>

### Table 8 Predicted measured error statistics before and after compensation

<table>
<thead>
<tr>
<th>Compensation method</th>
<th>Before compensation</th>
<th>( L_2 )</th>
<th>( L_\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (( \mu )m)</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Std. dev. (( \mu )m)</td>
<td>29.4</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>Max (( \mu )m)</td>
<td>56.3</td>
<td>3.5</td>
<td>2.7</td>
</tr>
<tr>
<td>Error reduction (%)</td>
<td>0.0</td>
<td>93.8</td>
<td>95.2</td>
</tr>
</tbody>
</table>
10 mm from the corner of the pocket. The accuracy improvements of these slot and pocket features confirm the accuracy of the estimated error parameters and efficacy of the calibration methodology using six artifact orientations.

5 Conclusions

A contact trigger probe measurement system and methodology for kinematic calibration of mMTs have been developed that are consistent with mMT value characteristics of small size, high accuracy, low cost, and portability. Experimental calibration of a three-axis mMT has been conducted using the system and methods developed. The calibration and compensation results have been validated by analyzing machined features. The following conclusions can be drawn from this work:

1. A flexure measurement system and planar artifact has been designed for mMT calibration. The small working envelope of an mMT allows for a small artifact, typically smaller than 100 mm in diameter.
2. A hybrid measurement approach using off-machine and on-machine measurements has been developed to increase the speed of calibration. The off-machine measurements only need to be taken once, since the repeatability of the kinematic coupling ensures that the artifact orientations are the repeatable for each recalibration. The on-machine measurements are needed for each recalibration, but can be executed quickly. Thus, recalibration is achievable in less than 30 minutes.
3. Off-machine measurements are conducted on a calibrated measurement device (CMD) to determine the relative orientations between the pallet coordinate system and each of the artifact orientations. On-machine probing measurements are taken using the developed measurement system for each artifact orientation. These measurements are analyzed to determine the machine tool error parameters.
4. The measurement error of the system employed here is dominated by the repeatability of the kinematic coupling. The standard deviation of the kinematic coupling repeatability is measured to be 0.58 μm, roughly ten times the standard deviation of the contact measurement. Improvement of the kinematic coupling repeatability is possible by polishing the vee grooves to reduce their surface roughness.
5. Increasing the number of artifact orientations from three to six greatly increases the accuracy of error prediction in the presence of measurement noise. Using six artifact orienta-

tions, the calibration accuracy increases with the number of measurement points. A minimum of 15 well-distributed measurements per orientation is desirable.

6. Experimental calibration using six artifact orientations and a zero-order error model compensates for more than 90% of the uncompensated error of the chosen three-axis mMT.

Acknowledgment

The authors thank the American Society for Engineering Education (ASEE) for support of this work in the form of a National Defense Science and Engineering Graduate Fellowship. The authors also gratefully acknowledge Boeing’s financial support of the UIUC micro-manufacturing program. This work was also supported by the University of Illinois NSF Nano-CEMMS Center under NSF Grant No. DMI-03-28162.

References
